"*Let there be h*"*!* An Existence Argument for Planck's Constant

by

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Abstract: Planck's constant *h* is considered to be a fundamental Universal constant of Physics. And although we can experimentally determine its value to great precision, the reason for its existence and what it really means is still a mystery. Quantum Mechanics has adapted it in its mathematical formalism, as it also has the Quantum Hypothesis. But QM does not explain its meaning or prove its existence. Why does the Universe need *h* and *energy quanta?* Why does the mathematical formalism of QM so accurately reflect physical phenomena and predict these with great precision? Ask any physicists and uniformly the answer is "that's how the Universe works". The units of *h* are in *energy-time* and the conventional interpretation of *h* is as a *quantum of action*. But in this brief note we take a different view. We interpret *h* as *the minimal accumulation of energy* that can be manifested in our measurements. Certainly the units of *h* agree with such interpretation. Based on this we provide a plausible explanation for the existence of Planck's constant, what it means and how it comes about. We show that the existence of *Planck's constant* is not so much dictated by the Universe but rather by Mathematics and the inner consistence and calibrations of Physics.

Introduction: In another note we discuss *The Interaction of Measurement*. We argue there that direct measurement of a physical quantity $E(t)$ involves a physical interaction between the 'source' of the quantity and the 'sensor' (the point where the interaction takes place). For measurement to be made an interval of time Δt must have lapsed and an incremental amount ΔE of the quantity will be absorbed by the 'sensor'. This absorption happens when there is an interaction equilibrium between the 'source' and the 'sensor'. At equilibrium, the 'average quantity from the source' will equal to the 'average quantity E_{av} at the sensor'. *Consider this. Nothing in our observable World can exist without time, when it* ***is*** *in equilibrium with its environment and its presence can be observed and measured.*

Furthermore it was shown at the same note that *The Interaction of Measurement*, i.e. the mathematical relationship between the quantity E_0 at the 'sensor' at time $t = 0$, the amount ΔE of the quantity absorbed by the 'sensor' at each interaction cycle, and the average quantity E_{av} at the 'sensor' during each interaction cycle, is given by *Planck's Formula.*

Mathematical Foundations: The following **mathematical equivalences** are proven elsewhere. We will use these in some of our arguments below:

I) $E(t) = E_0 e^{\nu t}$ *if and only if* $E_0 = \dfrac{\eta \nu}{e^{\eta \nu / \kappa \mathcal{T}_\eta} - 1}$ **(1)**

II) $E(t) = E_0 e^{\nu t}$ *if and only if* $\dfrac{\eta \nu}{e^{\eta \nu / \kappa \mathcal{T}_\eta} - 1}$ *is invariant with respect to t* **(2)**

III) $E(t) = E_0 e^{\nu t}$ *if and only if* $\Delta E = \eta \nu$ **(3)**

IV) *For any integrable function* $E(t)$, $\lim_{t \to 0} \dfrac{\eta \nu}{e^{\eta \nu / \kappa \mathcal{T}_\eta} - 1} = E_0$ **(4)**

where $\eta = \int_0^\tau E(u)du$ and $\mathcal{T}_\eta = \left(\dfrac{1}{\kappa}\right)\dfrac{\eta}{\tau}$ with κ an arbitrary scalar constant.

Using these mathematical results we were able to describe *The Interaction of Measurement* by the following **mathematical identity**: (An 'identity' is a tautology, A=A, always true. A 'mathematical identity' is an identity of pure mathematical quantities with no particular physical interpretation or physical law)

$$E_0 = \frac{\Delta E}{e^{\Delta E/E_{av}} - 1} = \frac{\eta \nu}{e^{\eta \nu / \kappa \mathcal{T}_\eta} - 1} \qquad \text{(if } E(t) = E_0 e^{\nu t}\text{)} \tag{5}$$

or,

$$E_0 \approx \frac{\Delta E}{e^{\Delta E/E_{av}} - 1} \approx \frac{\eta \nu}{e^{\eta \nu / \kappa \mathcal{T}_\eta} - 1} \qquad \text{(if } E(t) \text{ is integrable)} \tag{6}$$

Note: To avoid limits and approximations, we will assume identity **(5)** *in our discussions below.*

These purely mathematical results compare strikingly well with the mathematical form of

$$\textit{Planck's Formula:} \quad E_0 = \frac{h\nu}{e^{h\nu/kT} - 1} \tag{7}$$

In this comparison we have that $\eta = h$, $\mathcal{T}_\eta = T$ and $\kappa = k$ while $E(t) = E_0 e^{\nu t}$. **(8)**

The quantity $\eta = \int_0^\tau E(u)du$ plays a key role in the mathematical identity **(5)**. This can be viewed as the 'accumulation of $E(t)$ '. Likewise the quantity $\mathcal{T}_\eta$ can be viewed as 'temperature'. We make the following **description of 'temperature' of $E(t)$: A quantity $\mathcal{T}_\eta$ is 'temperature' if it is inversely proportional to the time τ for an accumulation η to occur. Thus if $\mathcal{T}_\eta$ is doubled, the accumulation will be twice as fast, and visa-versa.** This description of temperature agrees well with our physical sense of temperature. *We will assume that temperature have this property, no matter how otherwise it may be defined.*

For fixed η , we can define $\mathcal{T}_\eta = \left(\frac{1}{\kappa}\right)\frac{\eta}{\tau}$ which will be unique up to an arbitrary scalar constant κ .

Conversely, for a given $\mathcal{T}$ as described above, we will have $\mathcal{T} = \rho \cdot \frac{1}{\tau}$, where ρ is a 'proportionality constant'. By setting $\rho = \frac{\eta}{\kappa}$ we can write $\mathcal{T}$ as $\mathcal{T}_\eta = \left(\frac{1}{\kappa}\right)\frac{\eta}{\tau}$. We have,

Temperature Equivalence:

Given η , we have $\mathcal{T}_\eta = \left(\frac{1}{\kappa}\right)\frac{\eta}{\tau}$ where κ is some arbitrary scalar constant.

Conversely, given $\mathcal{T}$ as described above we have $\mathcal{T} = \mathcal{T}_\eta = \left(\frac{1}{\kappa}\right)\frac{\eta}{\tau}$ for some fixed η and arbitrary scalar constant κ .

Any temperature. therefore, will have some fixed η and arbitrary scalar constant κ associated with it.

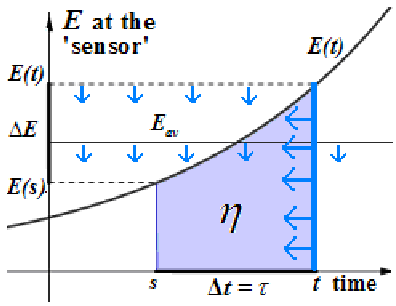

figure 1

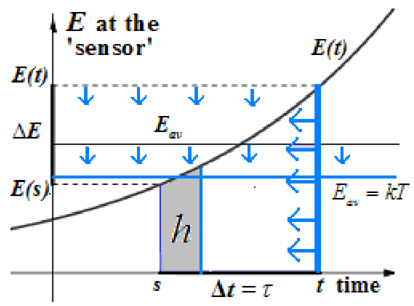

figure 2

$$E_0 = \frac{\Delta E}{e^{\Delta E/E_{av}} - 1} = \frac{\eta\,\nu}{e^{\eta\,\nu/\kappa\,T_\eta} - 1}\ ,\ \Delta E = \eta\,\nu\ ,E_{av} = \kappa\,T_\eta\ ,\ T_\eta = \left(\frac{1}{\kappa}\right)\frac{\eta}{\tau}\ ,\ E(s) = E_0\ ,E(t) = E_0 e^{\nu\,t}$$

From the mathematical equivalences **(1)** and **(2)** above we see that the 'accumulation' η can be *any* value and $\frac{\eta\,\nu}{e^{\eta\,\nu/\kappa\,T_\eta} - 1}$ will be invariant and will continue to equal to E_0. We can in essence (see *figure 1)* 'reduce' the formula $E_0 = \frac{\eta\,\nu}{e^{\eta\,\nu/\kappa\,T_\eta} - 1}$ by reducing the value of η and correspondingly the values of $E_{av} = \kappa\,T_\eta$ and $\Delta E = \eta\,\nu$, and visa versa. Thus we see that η and the corresponding $E_{av} = \kappa\,T_\eta$ go hand-in-hand to maintain the formula $E_0 = \frac{\eta\,\nu}{e^{\eta\,\nu/\kappa\,T_\eta} - 1}$ valid and invariant. These are *mathematical conclusions* true for any exponential function.

The Argument for the Existence of Planck's Constant: If $E(t)$ is energy, we would have that in the formula $E_0 = \frac{\eta\,\nu}{e^{\eta\,\nu/\kappa\,T_\eta} - 1}$ that describes the *Interaction of Measurement* the 'accumulation of energy' η can in essence be *any* value (but not 0 since in an interaction some time must lapse). The mathematical equivalences **(1)** and **(2)** above will allow η to be any value, and the average value $E_{av} = \kappa\,T_\eta$ will accordingly adjust keeping the formula true. Or we could set the value $E_{av} = \kappa\,T_\eta$ and then the value η will adjust keeping the formula invariant (see *figure 1*).

Thus, for a given $E(t)$ the value of η is determined by the value of E_{av}, and visa versa. And though the mathematical equivalences **(1)** and **(2)** above allow these values to be anything, the calibrations (theoretical and experimental) of these quantities in Physics restrict their value to be specific. If we let the quantity $E(t)$ in **(1)** above be energy, and let $\eta = h$ (Planck's constant) and $\kappa = k$ (Boltzmann's constant), then this will make $T_\eta = T$ (Kelvin temperature) (see *figure 2*). Or, if we start with $T_\eta = T$ and set the arbitrary constant $\kappa = k$, then this will force $\eta = h$. From the above **temperature equivalence** we will have some fixed accumulation η associated with Kelvin temperature. This accumulation of energy is calibrated to be *Planck's constant h.* Thus we see that *Planck's constant h, Boltzmann's constant k, and Kelvin temperature T* are so defined and calibrated to fit Planck's Formula. Planck's constant *h* exists and has this specific value because the average energy of a system (per degree of freedom) is given by *kT* with *k* having a specific calibrated value and *T* given is degrees Kelvin.

Conclusion: Planck's Formula is a mathematical identity that describes The Interaction of Measurement. It is invariant to time, accumulation of energy or amount of energy absorbed. Planck's constant exists because of this mathematical identity time-invariance. The calibration of Boltzmann's constant k and Kelvin temperature T, along with kT being the average energy, determine the specific value of Planck's constant h. It is the 'minimum accumulation of energy' that can be manifested by our measurements.

The argument in summary:

- *The Interaction of Measurement* (and more broadly *The Interaction of Energy)* is described by *Planck's Formula* which is an exact mathematical identity. Certain amount of time will lapse, an accumulation of energy will build up, and an amount of energy will be absorbed. All these quantities are related through *Planck's Formula.*

- We have the mathematical equivalence **(2)** which makes *Planck's Formula* invariant to the amount of time that will lapse, or the amount of energy that would accumulate or be absorbed. We can therefore in **(5)** let $\eta = h$ *, Planck's constant,* and set the arbitrary scalar $\kappa = k$ *, Boltzmann's constant.* Then the 'temperature' $\mathcal{T}_\eta$ will adjust to some value keeping the Formula valid. This value is $\mathcal{T}_\eta = T$ (degrees Kelvin). (see *figure 2*)

- Kelvin temperature *T* is that calibrated value (both theoretically and experimentally), along with Boltzmann's constant *k*, that results in an accumulation of energy equal to Planck's constant *h,* thus keeping Planck's Formula invariant and true.

- *Planck's constant* existence is a consequence of the time-invariance of a mathematical identity and the theoretical self-consistency of Physics, along with the system of calibrations used in Physics. Not some deep inner workings of some mysterious fundamental Universal truth.

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