Planck's Law is an Exact Mathematical Identity

by
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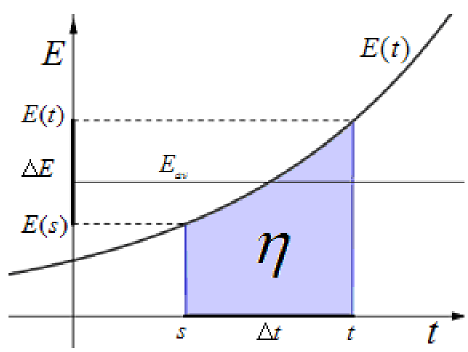
Abstract: In this short note we mathematically derive Planck's Formula for blackbody radiation in Quantum Physics. This derivation shows that under certain plausible assumptions Planck's Formula is an *exact mathematical identity that describes the interaction of energy*. Furthermore, these assumptions are well justified by mathematical derivations and established physical principles.

Notation:

$$\Delta E = E(t) - E(s)$$
$$\Delta t = \tau = t - s$$
$$E_{av} = \bar{E} = \frac{1}{t-s}\int_s^t E(u)du$$
$$\eta = P = \int_s^t E(u)du$$

Mathematical Identity: *For any integrable function* $E(t)$, $\eta = \int_s^{s+\eta/E_{av}} E(u)du$ (1)

proof: (see figure below)



Mathematical Derivations: *(the following are **proven elsewhere**)*

I) $E(t) = E_0 e^{\nu t}$ *if and only if* $E(s) = \dfrac{\eta\nu}{e^{\eta\nu/E_{av}} - 1}$ (2)

II) $E(t) = E_0 e^{\nu t}$ *if and only if* $\Delta E = \eta\nu$ (3)

III) *For any integrable function E(t),* $\lim_{t \to s} \dfrac{\eta\nu}{e^{\eta\nu/E_{av}} - 1} = E(s)$ (4)

Planck's Formula Derivation:

Planck's Formula for blackbody radiation states that, $E_0 = \dfrac{h\nu}{e^{h\nu/kT} - 1}$ (5)

where E_0 is the energy of radiation, ν is the frequency of radiation and T is the (Kelvin) temperature of the blackbody, while h is Planck's constant and k is Boltzmann's constant.

Planck's Formula as originally derived describes what physically happens at the 'source' (the blackbody). We'll consider instead what happens at the 'sensor' when making a measurement. We consider that measurement involves an 'interaction' between the 'source' and the 'sensor'. This interaction can be mathematically described as a functional relationship between $E(s)$, the energy locally at the 'sensor' at time s; ΔE, the energy absorbed by the 'sensor' making the measurement; and $\bar{E}$, the average energy at the 'sensor' during measurement. Note that *Planck's Formula* (5) has the same mathematical form as the mathematical equivalence (2) and as the limit (4) above. By letting $E(s)$ be an exponential, however, from (2) we get an *exact* formula, rather than the limit (4) if we assume that $E(s)$ is only an integrable function.

Assumptions:

1) Energy locally at the 'sensor' can be represented by $E(s) = E_0 e^{\nu s}$*, where* E_0 *is the intensity and* ν *is the frequency of radiation.*

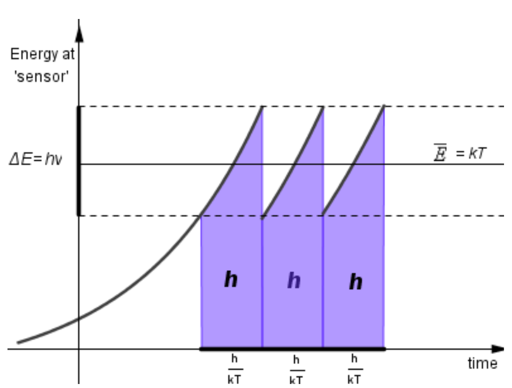
2) When measurement is made, the 'source' and the 'sensor' are in equilibrium, and so the 'average energy of the source' is equal to the 'average energy at the sensor'. We have $\bar{E} = kT$.

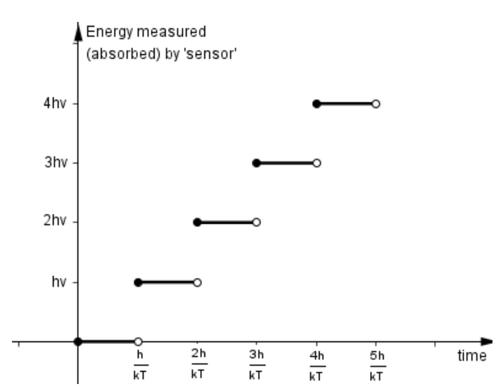
3) Planck's constant h represents the minimal amount of accumulated energy at the 'sensor' that can be measured. We have $\eta = h$.

Using the above *Mathematical Identity* and *Assumptions* we have Planck's Formula,

$$h = \int_0^{\frac{h}{kT}} E_0 e^{\nu u} du = \frac{E_0}{\nu}\left[e^{h\nu/kT} - 1 \right] \quad \text{and so, } E_0 = \frac{h\nu}{e^{h\nu/kT} - 1}.$$

From this formulation we see that Planck's Formula is an exact mathematical identity that describes the interaction of energy measurement. That is to say, it gives a mathematical relationship between the energy locally at the 'sensor', the energy absorbed by the 'sensor', and the average energy at the 'sensor' during measurement. Note further that when an amount of energy ΔE is absorbed by the 'sensor', the representation $E(s)$ 'collapses' (*see figures below)*





Conclusions:

1. Planck's Formula is an *exact mathematical identity* that describes the interaction of measurement.
2. Energy propagates continuously, but the absorption or measurement of energy is made in discrete *'equal size sips'*.
3. The absorption of energy is proportional to frequency, $\Delta E = h\nu$
4. Locally energy can be represented by $E(s) = E_0 e^{\nu s}$, where E_0 is the intensity and ν is the frequency of radiation.
5. The energy measured ΔE vs. Δt is linear with slope $\nu\, kT$ for constant temperature *T*.
6. The time Δt required for an accumulation of energy h to occur at temperature T is given by $\Delta t = \dfrac{h}{kT}$.

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