The Temperature of Radiation

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Abstract: Temperature as is formally defined in Thermodynamics just does not apply to radiation. Yet, as a physical property it seems that radiation should have temperature. In this short note we define the **temperature of radiation** and make plausible arguments that this definition is equivalent to Kelvin temperature as defined in Thermodynamics.

Definition of Temperature of Radiation:

Let $E(\mathbf{x},t)$ be radiation energy at point $\mathbf{x}$ and at time *t*. Consider an accumulation of energy η locally at point $\mathbf{x}$ over a duration of time τ. We have,

$$\eta = \int_0^\tau E(\mathbf{x},t)dt$$

Let

$$\mathcal{T} = \left(\frac{1}{\kappa}\right)\frac{\eta}{\tau}, \quad \text{where } \kappa \text{ is a scalar constant}$$

We define the **'temperature of radiation'** at point $\mathbf{x}$ to be $\mathcal{T}$. Note that when the radiation built up at $\mathbf{x}$ is faster, the value of $\mathcal{T}$ is higher, and when the built up is slower this value is lower. This is analogous to saying that if 'particles move faster' the temperature will be higher and if they move slower the temperature will be lower!

As a direct consequence of our definition here we have, $\bar{E} = \frac{1}{\tau}\int_0^\tau E(\mathbf{x},t)dt = \frac{\eta}{\tau} = \kappa\mathcal{T}$

Since κ is a constant, used for scale calibration, if we set $\kappa = k$, Boltzmann's constant, we have the average energy (locally at a point) to be,

$$\bar{E} = k\mathcal{T}$$

The average energy of a system (per degree of freedom) with temperature *T* (Kelvin) is known to be

$$\bar{E} = kT$$

Planck's Law connection:

In another paper we prove the following 'Planck-like' mathematical equivalence:

Theorem: $E(t) = E_0 e^{\nu t}$ **if and only if** $E_0 = \dfrac{\eta \nu}{e^{\eta \nu / \kappa \mathcal{T}} - 1}$,

where η and κ are arbitrary constants

If we let $\eta = h$ and $\kappa = k$, where *h* is Planck's constant and *k* is Boltzmann's constant, in the above 'Planck-like' equivalence we get, $E_0 = \dfrac{h\nu}{e^{h\nu / k\mathcal{T}} - 1}$

while Planck's Formula can be written as, $E_0 = \dfrac{h\nu}{e^{h\nu / kT} - 1}$.

Discussion:

These plausible comparisons suggests that $\mathcal{T} = T$. The definition of the 'temperature of radiation' given above then may be equivalent to Kelvin temperature *T* as defined in Thermodynamics. The notion of 'degrees of freedom' seems to be equivalent to 'locally at a point', while Planck's Law can be shown to be an *exact mathematical identity*. Furthermore, Planck's Law suggests a local (at a point) time-dependent representation of energy by an exponential $E(t) = E_0 e^{\nu t}$ where E_0 is the intensity and ν is the frequency of radiation.

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